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MICROGRAVIMETRY AND THE MEASUREMENT AND APPLICATION OF GRAVITY --ETC(U)
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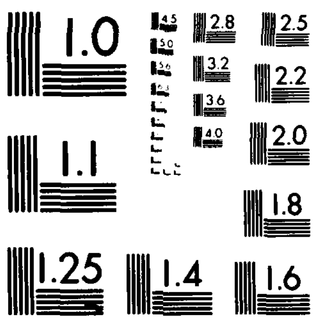

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MICROGRAVIMETRY AND THE MEASUREMENT AND APPLICATION OF GRAVITY GRADIENTS

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INTRODUCTION

Gravimetry is the science which studies the earth's gravitational field in all its aspects. Applied gravimetry involves measurements of the vertical component of the gravitational field g_z and the attempt to deduce geologic structure from the data. Any inversion of the gravity data to yield a possible causative mass (or density) distribution will be nonunique (as with inversions of all potential field data); however, by coupling geological constraints and other available geophysical data, the range of feasible solutions can be narrowly bracketed. Commonly, the objective of gravimetric exploration is the location of structural or stratigraphic environments typical of oil and gas or ore deposits. A perhaps less obvious application of gravimetry is to geotechnical problems, where the objective is shallow structural mapping to detect anomalous conditions such as subsurface cavities, fracture zones, faults, variation in depth to top of rock, buried river channels, etc. These high-resolution applications of gravimetry involve not only a significant scaling down in size and depth of the structures of interest and corresponding decrease in required profile and grid spacings, but also relative measurements of the acceleration of gravity in the μgal range (1 μgal = 10^{-6} gal = 10^{-6} cm/s^2 ≈ 10^{-9} times the normal gravitational acceleration). High-resolution gravimetry is properly referred to as microgravimetry (1-3). Application of microgravimetry, particularly to geotechnical problems, has enjoyed only limited success due to poor gravimeter sensitivity and accuracy and lack of appreciation for proper field procedures. However, the development of

a true "microgal" gravity meter* in the late 1960's has made microgravimetry a viable geotechnical field method.

Recognizing the potential value of microgravimetry for geotechnical applications, the U. S. Army Engineer Waterways Experiment Station (WES) began a research effort in 1978 to assess the benefits to the Corps of Engineers and WES of developing complete in-house capability for application of microgravimetric techniques. One of the most promising areas of application is the detection and delineation of subsurface cavity systems in areas with solution-susceptible bedrock. Microgravimetry has successfully been applied at a field test site in Florida to map a cavity system. Another area of interest and the subject of this paper is the use of microgravimetric techniques to determine the first vertical and horizontal derivatives (gradients) of g_z . The gravity gradients are of fundamental importance and offer three advantages over measurement of just g_z alone: (a) increased detectability limits and resolution of anomalies due to shallow structures; (b) gradient profiles have diagnostic properties which, in many cases, may make subsurface structure identification more straightforward; and (c) the gradients should selectively filter out anomalies from deeper structures and thus enhance the detection of anomalies due to shallow structures. The results of a controlled field study to evaluate techniques for determining gravity gradients and the use of these data for structural interpretations are presented in this paper.

GRAVITY GRADIENTS

Considering a Cartesian coordinate system (x,y,z), with the z-axis vertically downward, the derivatives of interest are $\partial g_z/\partial z$, $\partial g_z/\partial x$, and $\partial g_z/\partial y$. It is a natural approach to study a function such as g_z by examining its derivatives or gradients in specified directions. Since $g_z = \partial U/\partial z$, where U is the gravitational potential, we are interested in defining various components of the second derivative matrix $U_{,ij}$, where the comma indicates partial differentiation (with respect to the subsequent indices) and i and j = 1, 2, 3 corresponding to $x, y, z,$ respectively. For U due to a purely two-dimensional structure, $U_{,ij}$ is a second-order tensor. In particular, we are interested in $U_{,xz}$, $U_{,yz}$, and $U_{,zz}$. Since $\nabla^2 U = 0$ in source-free space, we know that $U_{,zz} = -(U_{,xx} + U_{,yy})$. Also since $\nabla^2 g_z = 0$, we have $g_{z,zz} = -(g_{z,xx} + g_{z,yy})$, which suggests a way in which $g_{z,zz}$ and $g_{z,z}$ (by integrating) could be obtained by numerical methods from gridded g_z data.

* LaCoste and Romberg, Inc., Model-D "Microgal" Gravimeter.

Indeed, numerous techniques for calculating $\partial g_z/\partial z$ ($g_{z,z}$) have been proposed; however, all suffer from all the inaccuracies and deficiencies of the original g_z data and from fictitious anomalies introduced by the numerical process (4). Clearly, the problems associated with calculating $g_{z,x}$ or $g_{z,y}$ from gridded g_z data should be much less difficult. However, the basic problem remains that, for standard survey techniques, the grid data are too widely spaced for the gradient values to have any real significance for delineating structures.

It is highly desirable that field techniques be developed for directly determining the gravity gradients. Since gravity gradiometers do not exist (5), the determinations will be finite difference approximations to the partial derivatives, i.e.

$$\lim_{\Delta z \to o} \Delta g_z/\Delta z \to \partial g_z/\partial z \;;\quad \lim_{\Delta x \to o} \Delta g_z/\Delta x \to \partial g_z/\partial x \;;\quad \text{etc.}$$

Vertical gradient

For the vertical gradient, the procedure simply involves measuring g_z on the surface and at one or more positions vertically above the surface station. Measurements in tall buildings on a floor-by-floor basis have been utilized (a) for the determination of the free-air vertical gradients and (b) to accurately calibrate gravimeters (6,7). Portable tower structures must be utilized in applications of the vertical gradient data (c) to correct field gravity data and (d) as an exploration method to locate anomalous masses and structures. Attempts to make practical field vertical gravity gradient measurements have met with considerable difficulty. The problem reduces to a trade-off between practical field implementation (manageable tower height), greater tower height (distance between measurement stations) to decrease the probable error in the determination, and the need to approximate the true gradient.

Horizontal gradient

For the determination of horizontal gradients, the situation is much simpler since virtually any horizontal separation between measuring stations is logistically feasible. The main consideration now becomes one of keeping the spacing small enough to adequately define the anomaly. Another consideration is the need to define the direction of the maximum horizontal gradient. This can be accomplished by double track profiling or by measuring at three locations at the corners of say a right triangle (13) for each gradient determination. For geotechnical applications, horizontal separations of the range of

5 to 10 m will be required, and this data will conveniently be available routinely in microgravimetric surveys.

Relation between gradients for
two-dimensional anomalous structures

While two-dimensional structures do not exist in nature, in many cases the approximation to two-dimensional conditions is very close. Thus, analytical consideration of two-dimensional structures (constant cross section and long in say the y-direction) is not only convenient but useful for a large number of real geologic conditions. For a two-dimensional anomalous structure, the vertical gradient $g_{z,z}(x,0)$ and horizontal gradient $g_{z,x}(x,0)$ on the surface are related by a Hilbert transform (14)

$$g_{z,z}(x,0) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g_{z,x}(\xi,0)}{\xi - x} \, d\xi$$

where x is the profile point and the integral is interpreted in the sense of its Cauchy principal value. This relation is of great value, since, if it proves impractical to determine either of the gradients in the field, one of the gradients can be calculated from the other. A computer program has been written to perform the Hilbert transform for discrete data.

RESULTS OF A FIELD STUDY OF GRAVITY GRADIENT TECHNIQUES

There has been much speculation on the feasibility and utility of gravity gradient determinations. However, no definitive study has been reported. The study reported here is a preliminary attempt at a definitive evaluation of gravity gradient techniques. Three criteria guided the selection of the test case: (a) the anomalous structure should be precisely defined; (b) the anomaly both in g_z and the gradients should be large (in a "microgravimetric sense") and should have a relatively short wavelength; and (c) the structure should approximate two-dimensional conditions. These criteria seemed most easily satisfied by a shallow man-made structure. The structure chosen was the concrete-lined drainage channel shown in Figure 1. Since the structure is at the surface, the gravity anomaly is large and has a short wavelength. Also, since the channel extends to either side of the bridge for at least 100 m with no significant change in cross section, the structure is approximately two-dimensional. The bridge itself is the only major non-two-dimensional

Figure 1. Drainage channel structure chosen for microgravity and gravity gradient field tests

aspect of the site. Pertinent dimensions are given in the diagram in Figure 2, which will also be the basis for two-dimensional model calculations.*

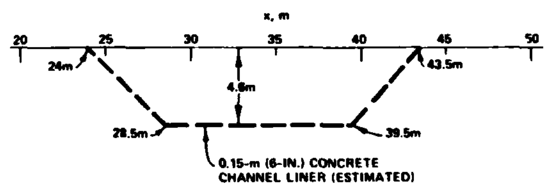

Figure 2. Location of drainage channel along profile line and dimensions of two-dimensional model. Profile line direction is N22°E

The survey over the drainage channel consisted of 16 stations over a 55-m line perpendicular to the channel, with the approximate center of the channel at the 34-m position. At each station g_z measurements were made at the ground surface and at a nominal elevation of 1.3 m vertically above the ground station. No elevation

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* Two-dimensional model calculations were accomplished using a computer program TALGRAD which utilizes the algorithm of Talwani (15) to compute g_z profiles due to an arbitrary number of polygonal cross-sectional structures. The program also allows for the computation of $\Delta g_z/\Delta z$ and $\Delta g_z/\Delta x$ along the profile for arbitrarily specified Δz and Δx .

or Bouguer corrections were necessary for the data (no elevation change). The data were corrected for latitude change in station location in the usual manner (16). Linear drift corrections were applied to the data utilizing base station reoccupations, and the drift curves were compared to theoretical earth tide curves calculated for the site to verify consistent gravimeter performance.

Results of the two-dimensional model calculations and the observed gravity data are compared in Figure 3. The model profile results agree closely in anomaly amplitude and width with the observed data, with the major deviation being the approximately 70-μgal positive anomaly (relative to the model profile) between 30 and 36 m. However, this is precisely where the non-two-dimensional aspects of the structure, i.e., the bridge pillars and beams, should make a positive contribution to the observed gravity profile. The maximum positive contribution due to the pillar beneath the profile line should be about 30 μgal, with the remainder of the 70-μgal anomaly accounted for by the other pillar and the bridge beams.

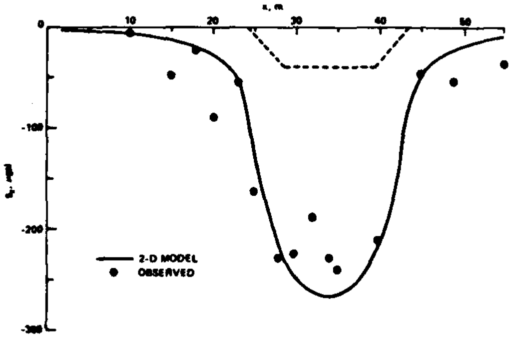

Figure 3. Gravity profiles across the drainage channel

Finite difference approximations to the horizontal and vertical gradients of g_z along the profile were computed for the two-dimensional model and from the observed field data. Figures 4 and 5 present the horizontal and vertical gradient profiles, respectively, for the two-dimensional model. For the horizontal gradient (Figure 4), profile values were computed for $\Delta x = 3$ m and $\Delta x = 10$ m. The finite difference approximation $\Delta g_z/\Delta x$ to $\partial g_z/\partial x$ should become better as Δx decreases; and clearly the horizontal gradient profile for $\Delta x = 3$ m is sharper and has greater amplitude than the profile for $\Delta x = 10$ m, as expected. The vertical gradient profile (Figure 5) was computed for $\Delta z = 1.3$ m, corresponding to the nominal value used for the field measurements. Note that the four corners of the structure are fairly well defined in Figure 5 by the vertical gradient profile.

Horizontal and vertical gradient profiles determined from the field data are shown in Figures 6 and 7. The horizontal gradient

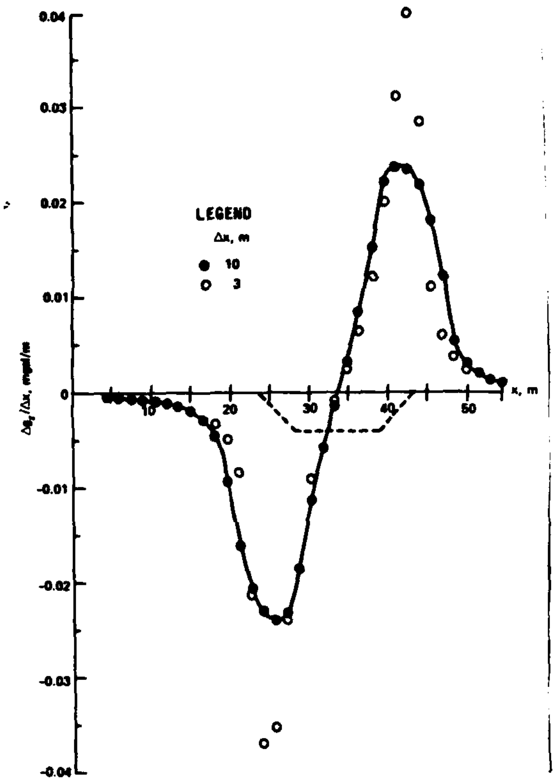

Figure 4. Analytical horizontal gradient profile for two-dimensional model of drainage channel (see Figure 2)

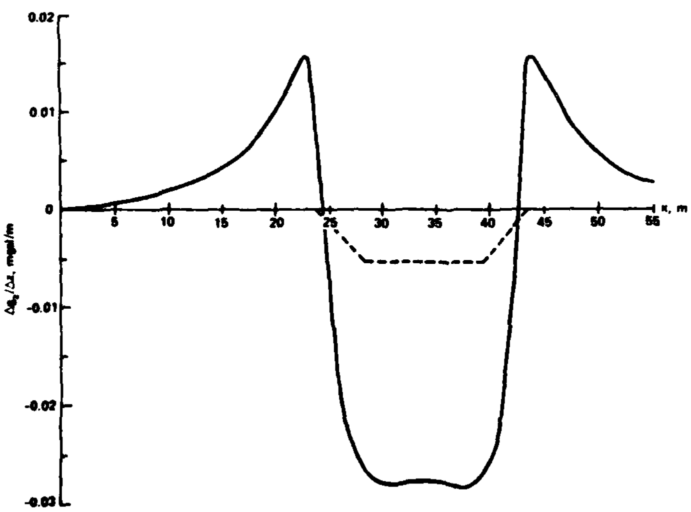

Figure 5. Analytical vertical gradient profile for two-dimensional model of drainage channel determined for $\Delta z = 1.3$ m

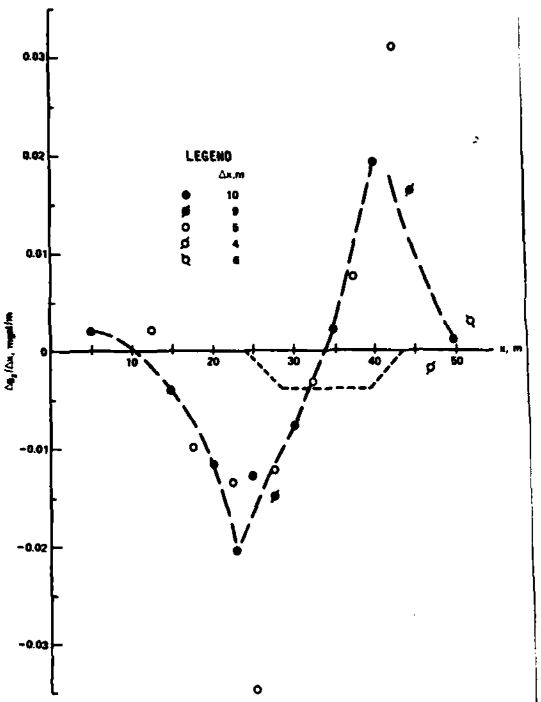

Figure 6. Horizontal gradient profile deduced from the gravity profile over the drainage channel (see Figure 3)

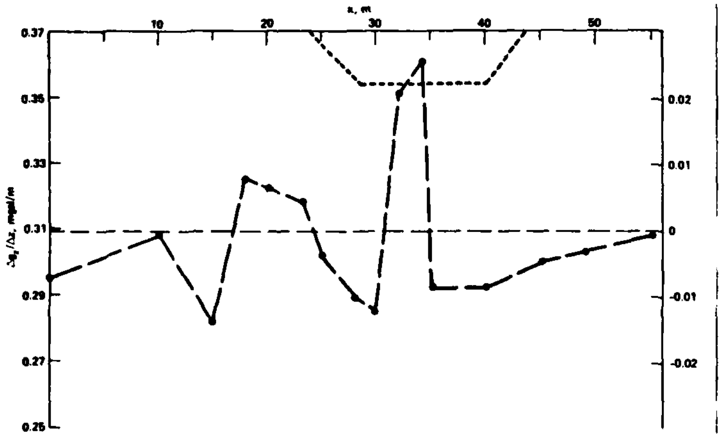

Figure 7. Vertical gradient profile deduced from portable-tower gravity survey over the drainage channel with $\Delta z \approx 1.3$ m

values are shown for various values of Δx (corresponding to various possible combinations of stations); however, the dashed line connects points for $\Delta x = 10$ m. The comparison between Figures 4 and 6 is quite good for the curves for $\Delta x = 10$ m, both in general shape and in anomaly amplitude and width. Also, the general trend of increasing anomaly amplitude and sharpness with decreasing Δx is seen in the observed data in Figure 6. The vertical gradient data (Figure 7) are very erratic and only with a great amount of smoothing and imagination do the results resemble the model results of Figure 5. There are several possible reasons for the erratic nature of the vertical gradient data:

a. The value $\Delta z \approx 1.3$ m is too small for the vertical separation between measuring stations due to the probable error in the measurements, i.e., a larger Δz would result in a larger Δg_z and hence decrease the significance of the probable error.

b. The vertical gradient determination is more strongly affected by the non-two-dimensional aspects of the structure than the horizontal gradient determination.

c. The vertical gradient is known to be strongly influenced by very shallow density fluctuations, so a preferable procedure might be to make the lower g_z measurement some small distance, say 0.2 m or so, above the ground surface.

It is probable that the two large positive values of vertical gradient between 30 and 35 m are due to the bridge pillars and beams.

Calculation of vertical gradient profile from the horizontal gradient profile

Utilizing the horizontal gradient profile for the two-dimensional model (Figure 4), the vertical gradient profile shown in Figure 8 was computed using the Hilbert transform relation and computer program discussed earlier. Except for the sign reversal (caused by assuming the z-axis vertically downward for the two-dimensional model results), the vertical gradient profiles in Figures 5 and 8 agree qualitatively very well. The lower amplitudes and frequency content of the profile computed by the Hilbert transform are not unexpected; however, the comparison would improve if the profile itself were longer and/or a smaller Δx were used for the horizontal gradient profile.

Similarily, a vertical gradient profile (Figure 9) was

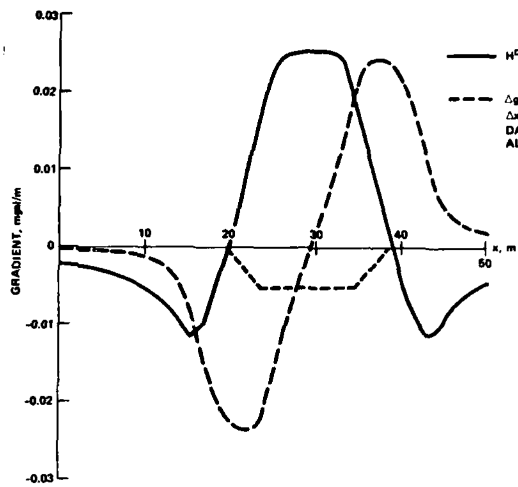

Figure 8. Vertical gradient profile calculated from horizontal gradient profile for the two-dimensional model by the Hilbert transform

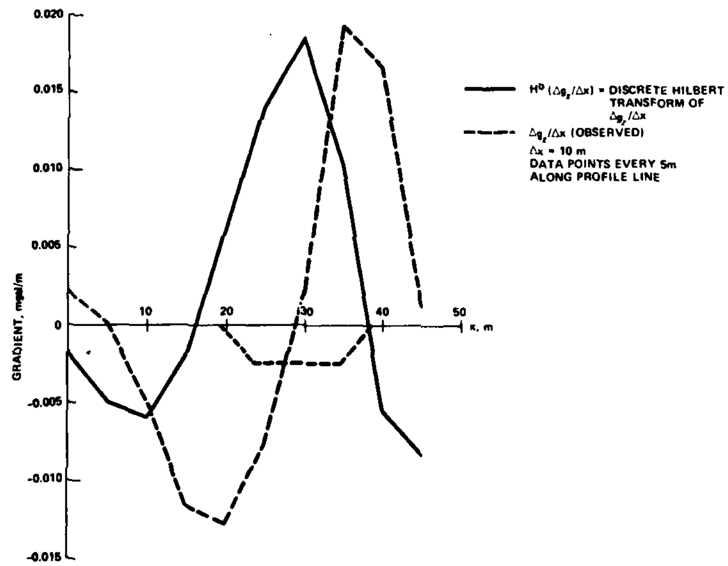

Figure 9. Vertical gradient profile calculated from the observed horizontal gradient profile over the drainage channel by the Hilbert transform (x = 0 is at 5 m of actual survey line)

computed from the horizontal gradient profile of the observed data (Figure 6) over the structure using the Hilbert transform. Again, only with extreme smoothing is there even qualitative similarity between the observed and calculated vertical gradient profiles (Figures 7 and 9, respectively). However, the profile in Figure 9 compares qualitatively quite well with the vertical gradient profiles in Figures 4 and 6. Thus, the procedure of determining the horizontal gradient profile from field measurements and then calculating the vertical gradient profile via the Hilbert transform appears very promising.

Utilization of gravity gradients

The motivations for determining gravity gradients have been discussed previously. A complete discussion of the possibilities for utilization of gradient data for subsurface structural delineation is beyond the scope of this paper and also premature. Thus, the concepts under consideration will only be briefly covered.

A very promising technique for displaying the gradient data is a gradient space plot, i.e., $g_{z,z}$ versus $g_{z,x}$. In such a space, each point will correspond to a given profile position or value of x. As an example, the gradient profile data for the two-dimensional model (Figures 4 and 5) result in the gradient space plot in Figure 10. Corresponding points along the profile and the plot are indicated, and the manner in which the geometry of the structure

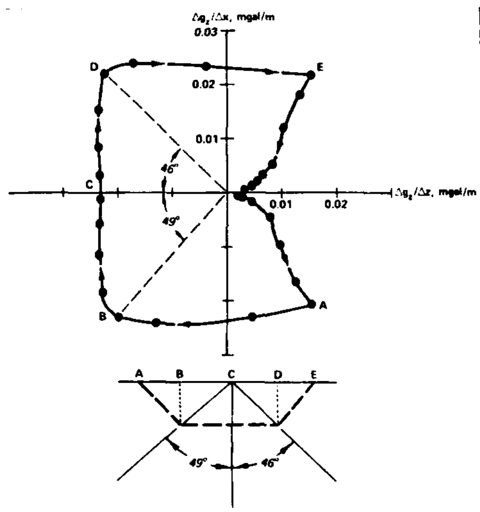

Figure 10. Gradient space plot for two-dimensional model of drainage channel

might be deduced is indicated.

Another promising technique involves the concept of the analytic signal along the profile, defined by $A(x) = g_{z,x}(x) - ig_{z,z}(x)$. Note that the gradient space plot is just a complex plane plot of the components of $A(x)$. The amplitude of $A(x)$ is defined in the usual manner, $a(x) = |A(x)| = (g^2_{z,x} + g^2_{z,z})^{1/2}$. Above a two-dimensional structure with corners, the $a(x)$ will be the superposition of symmetrical, bell-shaped curves, one for each of the corners. The properties of the bell-shaped curves determine the profile position and depth of the structural corner causing the signal. Thus, the decomposition of the $a(x)$ signal into bell-shaped curves represents in principle the solution of the structural problem.

SUMMARY AND CONCLUSIONS

Microgravimetry has been successfully applied both to a natural cavity site in Florida and to a man-made structure to delineate small-scale, shallow subsurface features. The horizontal gravity gradient profile has been adequately determined from a microgravimetric survey and successfully compared with the results of a two-dimensional model study. Measurement of the vertical gradient profile with a relatively short tower structure ($\Delta z = 1.3$ m) was not as successful. However, utilization of the Hilbert transform allows the vertical gradient profile to be calculated from the horizontal gradient profile. For cases in which the assumption of a two-dimensional, polygonal cross-sectional geometry is approximately valid, use of the gradient profiles permits a unique structural interpretation.

Future work in this research effort will concentrate on: (a) improved field procedures for microgravimetric surveys; (b) continued attempts to determine vertical gradient profiles across known structures using larger values of Δz than in the past; (c) further study of the application of the discrete Hilbert transform; and (d) in-depth studies of interpretative methods using the gradient data.

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